As a second example, consider a bandpass filter with a center frequency of 1000 MHz, a minimum 3-dB bandwidth of 100 percent, and a 20-dB rejection from at least 133 to 1867 MHz. The ripple is to be less than 0.05 dB.

We easily calculate the resonator length as 7.5 cm. The 3-dB frequency is calculated as $q=0.707$, and the 20-dB frequency as $q=0.98$. Examination of the curves shows that a three-resonator filter with $K=0.8$ (62.5-ohm stubs) will satisfy the requirements.

Many other possibilities are open. For example, the designer may examine the curves to see if the characteristics of his filter may be satisfied by the filters considered, and use this design instead of one which may be "over-designed." It must be determined if maximally flat or equal ripple response is really required. The filters also have bandstop properties, depending on the definition of center frequency.

These filters are not "optimum" designs in the sense of minimum number of resonators, best skirt response, etc. They are easily designed devices which, when they can be used, are quite functional.

To verify the theoretical results, a three-resonator quarter-wave shorted-stub coaxial filter was tested. The normalized characteristic admittance of the stub (K) was unity. The filter center frequency was 4700 MHz and its 3-dB bandwidth was 6400 MHz. Experimental and theoretical curves are shown in Fig. 5. The agreement is very close. The midband loss of the filter tested was about 0.2 dB, and the zero insertion loss design was shown to be valid.

In conclusion, we may say that the insertion loss versus frequency characteristics have been calculated for a class of equal stub admittance filters. Analytical expressions have been derived for any number of resonators, and graphs of the results have been prepared for from one to eight resonators. It was shown how the information presented enables the systematic design of filters of this class on the basis of insertion loss. It was also shown how bandwidth, admittance, etc., may be rigorously determined for specific cases of interest. The insertion loss curves are presented along with information as to their use and other pertinent theoretical material. Experimental verification of the theory has been presented. Further work along these lines that could be considered would concern other stub and line lengths, loss, time delay, and the analysis of transient response.

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On the Design of Stepped Transmission-Line Transformers

***Abstract*—The problem of matching a complex load impedance to a given transmission line using a series matching transformer is considered with a view to minimizing the transformer length and overall insertion loss. A graphical technique is presented that leads to a solution for the length and characteristic impedance of the transformer section. It is shown that this transformer reduces to the usual quarter-wave transformer for the particular case where the load is purely resistive. The performance of the transformer is also compared with the quarter-wave transformer for the case of a complex load impedance and a numerical example is given. It is shown that the design procedure is relatively simple and may lead to a significant reduction in the overall length and insertion loss of the matching section. While there is no significant improvement in the bandwidth for frequency-dependent loads, the proposed design still offers attractive features for matching transmitting antennas.**

Present techniques for matching a radio-frequency transmission line to a given complex load impedance are based on cancelling the input reactance (or susceptance) of the load, as seen from a pair of available terminals, and transforming the real part to that of the given line. Common examples that employ this conjugate matching technique are the open- and short-circuited parallel stubs and the quarter-wave series transformer. The design procedure for single and multiple sections of these devices have been amply discussed in the literature [1]–[3].

The purpose of this correspondence is to present a simple graphical method for designing a single transmission-line cable suitable for matching a complex load impedance at a single frequency. To show this we consider the situation in Fig. 1 where the matching section of unknown length d and characteristic impedance Z_0' is inserted between the load Z_L and the feed line. The input impedance Z_s at the junction of the two lines is given by the relation [1],

$$Z_s = Z_0' \frac{Z_L + Z_0' \tanh(\gamma d)}{Z_L' + Z_L \tanh(\gamma d)}, \tag{1}$$

where the propagation constant γ is the sum of the attenuation constant α and the phase constant β. Equating the real terms to R and the imaginary terms to zero, we obtain after some simplification:

$$Z_0^2 = r - \frac{x^2}{1-r} \tag{2}$$

$$y^2 = \left(\frac{1-r^2}{x}\right)\left(r - \frac{x^2}{1-r}\right), \tag{3}$$

where $y=\tan(\beta d)$ for $\alpha \to 0$, $r=R_L/R$, $x=x_L/R$, $Z_0=Z_0'/R$, and $\beta=2\pi/\lambda$. Equation (3) may be rewritten in the more convenient form:

$$\frac{(1-r)^2 r}{[r-(1+y^2)]} + x^2 = 0. \tag{3a}$$

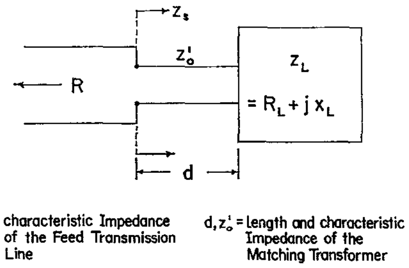

Fig. 1. Schematic diagram of the matched load Z_L.

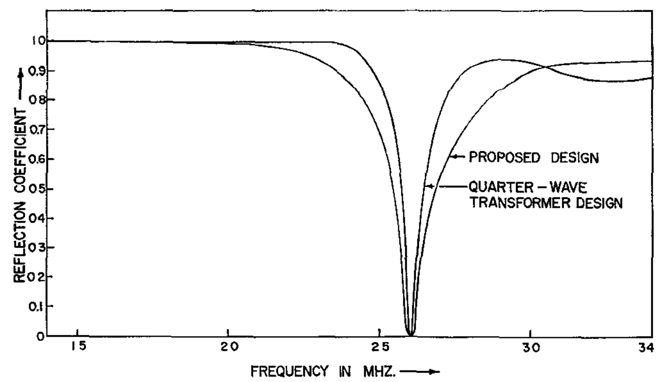

Fig. 2. Reflection coefficient versus frequency.

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Equations (2) and (3a) are sufficient to describe families of curves for various values of Z_0 and y as a function of r and x as circles and lemniscates, respectively. A solution for the parameters y and Z_0 is restricted to all values of r and x that lie inside the $Z_0=0$ and $Z_0=\infty$ circles in order to make the desired value of Z_0 positive and real. It is interesting to note that the parameter y is infinite for purely resistive loads leading to the usual quarter-wave transformer design. For complex load impedances, however, the overall length of the transformer is generally less than quarter wavelength when the range of y is between zero and infinity.

The proposed design procedure may therefore be summarized by the following steps.

1) Normalize R_L and X_L with respect to R to obtain r and x.
2) Locate the normalized impedance point and find the particular circle and lemniscate that intersect at the same point noting the labeled values of Z_0 and y.
3) Compute Z_0' and d. If the resulting value of d is negative, add a half wavelength section of characteristic impedance Z_0'.
4) If the impedance point in step 2) lies outside the $Z_0=0$ or $Z_0=\infty$ circles, a section of characteristic impedance R must be inserted between the load and the transformer. The length of this section is obtained by moving a minimum distance towards the generator, along a constant VSWR circle, to a new load point located inside the $Z_0=0$ or $Z_0=\infty$ circles. The remaining procedure is identical to steps 2) and 3).

The advantage of the proposed design over the quarter-wave transformer is the possible reduction in the insertion loss which is primarily due to the smaller number of junctions and shorter length of the matching section. To illustrate this advantage in a practical case, a complex load impedance of $40.7-j220$ ohms, corresponding to a 45.72-meter dipole with length-to-diameter ratio of 1800 and operating at 2.6 MHz, was selected. The feed-line characteristic impedance was 30 ohms and the values of d and Z_0' were found to be 9.923 meters and 370 ohms, respectively. The quarter-wave design required a 30-ohm, 26.434-meter-long line in series with the load and a 28.846-meter-long quarter-wave section with a characteristic impedance of 4.6 ohms. The reduction in the insertion loss is apparent from the shorter length required. However, the narrow bandwidth for both transformers, as shown for this case in Fig. 2, may not be a serious disadvantage where, for instance, the dipole corresponds to a transmitting antenna.

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Nonsymmetrical Coupled Lines of Reentrant Cross Section

Frequently, UHF and microwave TEM components require tightly coupled transmission lines, directional couplers [1] of greater than −8 dB coupling, narrow-to-moderate-bandwidth bandpass filters [2], moderate-to-wide-bandwidth stopband filters [3], and 90-degree phase shifters [4], [5] are some examples. The novel geometry of the directional coupler with reentrant cross section [6]–[9] can result in very tightly coupled transmission lines and has been proved practical and useful. Recently, the basic geometry and the corresponding design equations for directional couplers with reentrant cross section were extended [10] to provide a slightly more general geometry capable of even greater coupling with practical physical dimensions. However, in the aforementioned cases, only symmetrical geometries and their related equations were presented. In this correspondence, the basic reentrant cross-sectional geometry and associated equations are generalized to include the nonsymmetrical case which is important for nonsymmetrical directional couplers [11] and coupled strip transmission-line filters requiring coupled nonsymmetrical lines [2].

Nonsymmetrical coupled lines of reentrant cross section are shown in coaxial, stripline, and microstrip form in Fig. 1(a), (b), and (c), respectively. The geometries of Fig. 1(a) and (b) were previously presented in symmetrical form [6]–[9]; the microstrip geometry of Fig. 1(c) is new and is presented here in the general nonsymmetrical form.

Application of the coupled-lines configuration to a specific problem requires specification of the even- and odd-mode admittances (or capacitances) of the coupled lines [1]. For line a, these admittances are denoted by Y_{oe}^a and Y_{oo}^a, respectively; for line b, they are denoted by Y_{oe}^b and Y_{oo}^b, respectively. These parameters are generally determined from a set of design equations for the particular problem at hand. The admittances are related to the normalized capacitances by

$$C/\varepsilon = \frac{376.7Y}{\sqrt{\varepsilon_r}}, \tag{1}$$

where

1) C/ε = normalized capacitance, which is dimensionless and independent of the permittivity of the medium;
2) Y = the unnormalized admittance in mhos;
3) ε_r = the relative dielectric constant of the medium;
4) ε = the permittivity of the medium in farads/unit length.

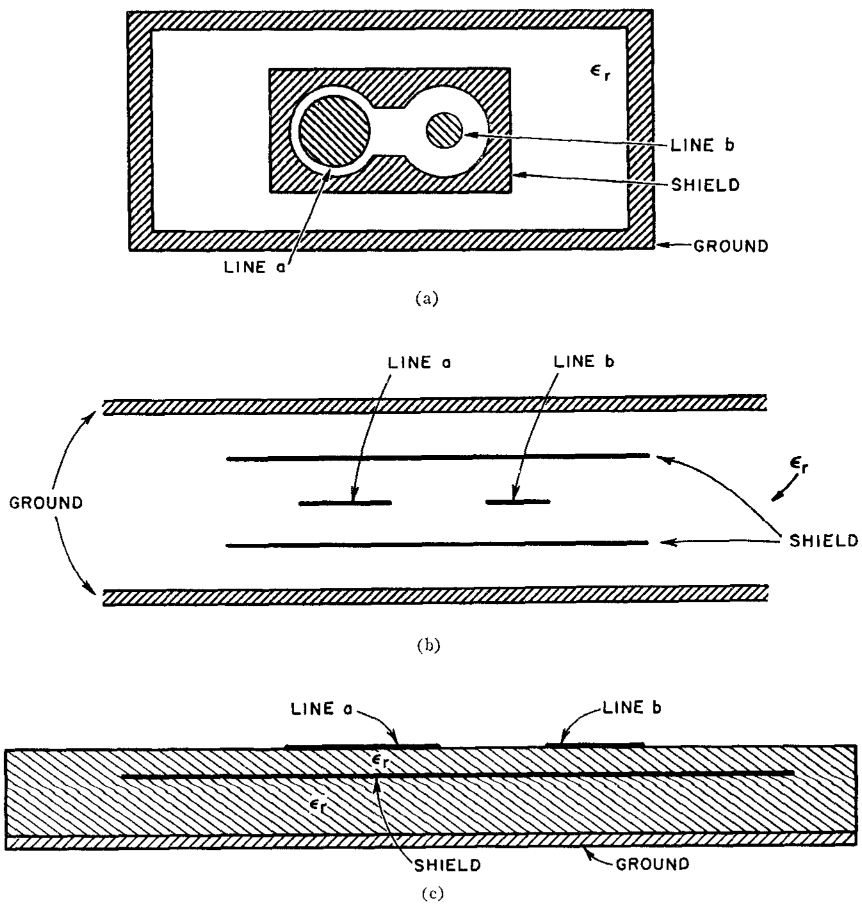

Fig 1. Nonsymmetrical coupled lines of reentrant cross section in (a) coaxial line, (b) strip line, and (c) microstrip.

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